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EFFECT OF CONDUCTION ON COMBINED FREE AND FORCED CONVECTION IN A VENTILATED CAVITY WITH A HEAT-GENERATING SOLID CIRCULAR BODY

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ABSTRACT
Numerical simulations of the effect of conduction on combined free and forced convection (mixed convection) heat transfer and fluid flow have been performed in a 2-D ventilated cavity with a finite size and finite conductivity solid circular body. The wall of the cavity is assumed to be adiabatic. Flows are imposed at the bottom of the left wall and exit at the top of the right wall of the cavity. The heat generating body is placed at the center of the cavity. The present study simulates a practical system such as air-cooled electronic equipment with a heat component. The developed mathematical model is governed by the coupled equations of mass, momentum and energy and is solved by employing Galerkin weighted residual finite element method. The computation is carried out for wide ranges of Reynolds number, Richardson number. Various results such as the streamlines, isotherms, heat transfer rates in terms of the average Nusselt number and average fluid temperature in the cavity are presented for different parameters. The results indicate that both flow field and temperature distribution strongly depend on Reynolds number, Richardson number. It is also observed that the mentioned parameters have significant effect on average Nusselt number at the heated surface and average fluid temperature in the cavity.

Keywords: Solid Circular Body, Ventilated Cavity, Mixed Convection And Finite Element Method.

1. INTRODUCTION
Modification of heat transfer in cavities due to introduction of obstacles, partitions and fins attached to the wall(s) has received some consideration in recent years. Many authors have recently studied heat transfer with obstacles, partitions and fins, thereby altering the convection flow phenomenon.

Three related studies of mixed convection in a partially divided rectangular enclosure were respectively carried out by Hsu et al. [1], How and Hsu [2] and Calmidi and Mahajan [3]. The simulation was conducted for wide range of Reynolds and Grashof numbers. They indicated that the average Nusselt number and the dimensionless surface temperature depend on the location and height of the divider. Combined free and forced convection in a square enclosure with heat conducting body and a finite-size heat source was simulated numerically by Hsu and How [4]. They concluded that both the heat transfer coefficient and the dimensionless temperature in the body center strongly depend on the configurations of the system. Shuja et al. [5] numerically studied mixed convection in a square cavity due to heat generating rectangular body and investigated the effect of exit port locations on the heat transfer characteristics and irreversibility generation in the cavity. They showed that the normalized irreversibility increases as the exit port location number increases and the heat transfer from the solid body enhanced while the irreversibility reduces. The same authors considered heat transfer enhancement due to flow over a two-dimensional rectangular protruding bluff body [6]. Hung and Fu [7] studied the passive enhancement of mixed convection heat transfer in a horizontal channel with inner rectangular blocks by geometric modification. Unsteady mixed convection in a horizontal channel containing heated blocks on its lower wall was studied numerically by Najam et al. [8]. Tsay et al. [9] rigorously investigated the thermal and hydrodynamic interactions among the surface-mounted heated blocks and baffles in a duct flow mixed convection. They focused particularly on the effects of the height of baffle, distance between the heated blocks, baffle and number of baffles on the flow structure and heat transfer characteristics for the system at various Re and Gr/Re^2. Turki et al. [10] conducted a numerical investigation to analyze the unsteady flow field and heat transfer characteristics in a horizontal channel with a built-in heated square cylinder. They examined the effects of the blockage ratio, the Reynolds number and Richardson numbers on aerodynamic and heat transfer characteristics. Chang and Shiau [11] numerically investigated the effects of a horizontal baffle on the heat transfer characteristics of pulsating opposing mixed convection in a parallel vertical open channel. Bhoite et al. [12] studied numerically the problem of mixed convection flow and heat transfer in a shallow enclosure with a series of block-like heat generating component for a range of Reynolds and Grashof numbers and block-to-fluid thermal conductivity ratios. They showed that higher Reynolds number tend to create a recirculation region of increasing strength at the core region and the effect of buoyancy becomes insignificant beyond a Reynolds number of typically 600, and the thermal conductivity ratio has a negligible effect on the velocity fields. Recently Rahman et al. [13] studied of mixed convection in a square cavity with a heat conducting square cylinder at different locations. At the same time Rahman et al. [14] studied mixed convection in a vented square cavity with a heat conducting horizontal solid circular cylinder. Very recently Rahman et al. [15] analyzed mixed convection in a rectangular cavity with a heat conducting horizontal circular cylinder by using finite element method.

The purpose of this study is to examine the effect of a heat generating circular body on mixed convection in a square cavity. Numerical solutions are obtained over a wide range of Richardson numbers and Reynolds number. The dependence of the thermal and flow fields on the Richardson numbers and Reynolds number is studied in detail.

2. MODEL SPECIFICATION

The schematic of the system considered in this paper is shown in Fig. 1. The system consists of a square cavity with sides of length *L*, within which a heat generating solid circular body with diameter of d is centered. The cylinder body has a thermal conductivity of k_s and generates uniform heat per unit volume of Q. The side walls of the cavity are assumed to be adiabatic. It is assumed that the incoming flow is at a uniform velocity, u_i and at the ambient temperature, θ_i. An inflow opening located on the bottom of the left vertical wall, whereas the out flow opening at the top of the opposite side wall and the size of the inlet port is the same size as the exit port which is equal to $w = 0.1L$.

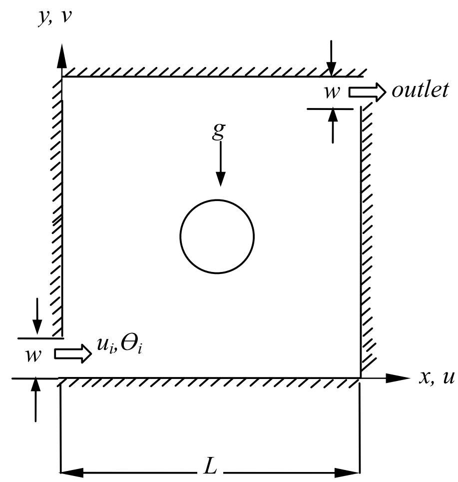


Fig. 1. Schematic diagram of the problem considered and coordinate system

3. MATHEMATICAL FORMULATION

The flow within the cavity is assumed to be two-dimensional, steady, laminar, incompressible and the fluid properties are to be constant. The radiation effects are taken as negligible and the Boussinesq approximation is used. The dimensionless equations describing the flow are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\theta \qquad (3)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{RePr}}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

For solid cylinder the energy equation is

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} + Q = 0 \qquad (5)$$

Here $\mathrm{Gr} = \dfrac{\beta g q L^4}{\upsilon^2 k}$ is the Grashof number, $\mathrm{Pr} = \dfrac{\upsilon}{\alpha}$ is the Prandtl number, $\mathrm{Re} = \dfrac{u_i L}{\upsilon}$ is the Reynolds number, $\mathrm{Ri} = \dfrac{\mathrm{Gr}}{\mathrm{Re}^2}$ is the Richardson number and Q is the heat generating parameter.

The above equations were non dimensionalized by defining

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{u_i},\ V = \frac{v}{u_i},\ P = \frac{p}{\rho u_i^2},$$

$$\theta = \frac{(T - T_i)}{(T_h - T_i)},\ \theta_s = \frac{(T_s - T_i)}{(T_h - T_i)}$$

Where X and Y are the coordinates varying along horizontal and vertical directions, respectively, U and V are, the velocity components in the X and Y directions, respectively, θ is the dimensionless temperature and P is the dimensionless pressure.

The boundary conditions for the present problem are specified as follows:

At the Inlet: $U = 1$, $V = 0$, $\theta = -0.5$

At the outlet: Convective boundary condition $P = 0$

At all solid boundaries: $U = 0, V = 0$

At the cavity walls: $\frac{\partial \theta}{\partial N} = 0$

At the fluid-solid interface: $\left(\frac{\partial \theta}{\partial N}\right)_{fluid} = K\left(\frac{\partial \theta_s}{\partial N}\right)_{solid}$

Where N is the non-dimensional distances either X or Y direction acting normal to the surface and K is the dimensionless ratio of the thermal conductivity $\left(K_s / K_f\right)$

The average Nusselt number at the heated surface is calculated as

$$Nu = -\int_0^{L_H} \frac{\partial \theta}{\partial X} dY$$

and the average temperature of the fluid is defined as $\theta_{av} = \int \theta d\overline{V} / \overline{V}$

where L_H is the length of the heated surface and $\overline{V}$ is the cavity volume.

4. COMPUTATIONAL PROCEDURE

The numerical procedure used in this work is based on the Galerkin weighted residual method of finite element formulation. In this method, the solution domain is discretized into finite element meshes, which are composed of triangular elements. Then the nonlinear governing partial differential equations i.e., mass, momentum and energy equations are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss quadrature method. Then the nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method. More details are available in Rahman *et al.* [15]

Geometry studied in this paper is an obstructed cavity; therefore several grid size sensitivity tests were conducted in this geometry to determine the sufficiency of the mesh scheme and to ensure that the solutions are grid independent. This is obtained when numerical results of the average Nusselt number *Nu*, average temperature θ_{av} and solution time become grid size independent, although we continue the refinement of the mesh grid. As can be seen in Table 1, five different non-uniform grids with the following number of nodes and elements were considered for the grid refinement tests: 24545 nodes, 3788 elements; 29321 nodes, 5900 elements; 37787 nodes, 5900 elements; 38163 nodes, 5962 elements and 48030 nodes, 7516 elements. As is shown in Table 1, 38163 nodes and 5962 elements can be chosen throughout the simulation to optimize the relation between the accuracy required and the computing time.

Table 1: Grid Sensitivity Check at $Re = 100$, $Ri = 1.0$ and $Pr = 0.71$

Nodes (elements)	24545 (3788)	29321 (4556)	37787 (5900)	38163 (5962)	48030 (7516)
Nu	4.46011	4.46014	4.46034	4.46044	4.46149
θ_{av}	0.57855	0.57845	0.57825	0.57815	0.57805
Time(sec)	323.610	408.859	563.203	588.390	793.125

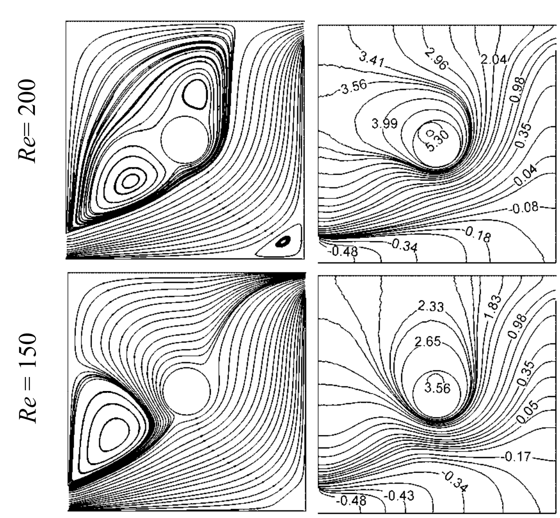
The present code was extensively exercised on the problem of House et al. [16] to check its validity. Table 2 compares the present results with the results by House et al. for Rayleigh number, $Ra = 0.0$, 10^5 and two values of $K = 0.2$ and 5.0. The present results have an excellent agreement with the results obtained by House et al.

Table 2: Nusselt Number Comparison for Pr = 0.71

Ra	k	Nu		
		Present work	House et al. [16]	Error (%)
0	0.2	0.7071	0.7063	0.11
0	1.0	1.0000	1.0000	0.00
0	5.0	1.4142	1.4125	0.12
10^5	0.2	4.6237	4.6239	0.00
10^5	1.0	4.5037	4.5061	0.00
10^5	5.0	4.3190	4.3249	0.14

5. RESULTS AND DISCUSSION

The effect of conduction on mixed convection flow in a ventilated square cavity having a heat-generating circular body is tested using a numerical technique. Different governing parameters are used as Reynolds number *Re*, Richardson number *Ri*, Prandtl number *Pr*, solid fluid thermal conductivity ratio *K* and heat generating parameter *Q*. Two significant parameters such as Reynolds number *Re*, Richardson number *Ri*, are investigated here for a ventilated square cavity. These



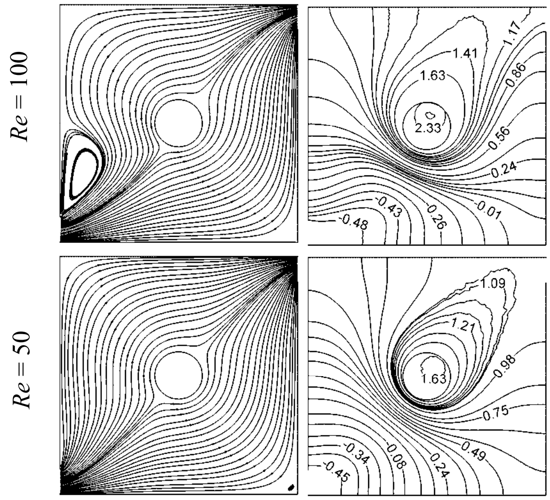


Fig 2. Streamlines and Isotherms for different values Re, at $Ri = 0.0$.

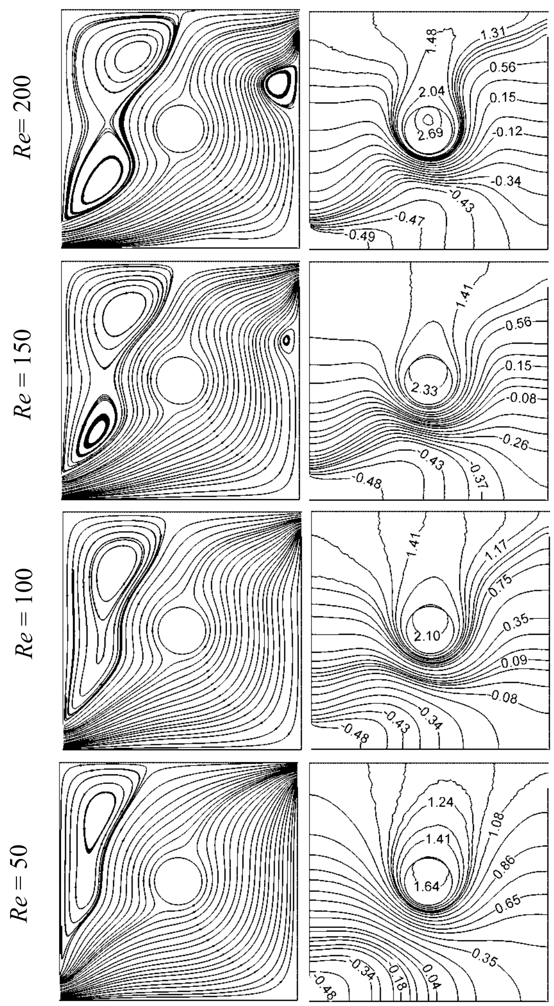


Fig. 3**:** Streamlines and Isotherms for different values Re, at $Ri = 1.0$.

ranges are varied as to $50 \leq Re \leq 200$, and $0.0 \leq Ri \leq 5.0$, while the other parameters K, Pr and Q are keeping fixed at 5.0, 0.71 and 5.0 respectively.

The effect of Re and Ri on streamlines and isotherms are shown in Figs. 2-4. The flow structure in the absence of the free convection effect ($Ri = 0.0$) is presented in the left column of the Fig. 2. For $Ri = 0.0$ and $Re = 50.0$, the induced flow enters into the cavity through small inlet and sudden expansion of the fluid is occurred in the cavity due to pressure rise. Thus the fluid occupies the whole cavity and bifurcates near the cylinder. It is also clear that the streamlines are symmetrical about the line joining the inlet and outlet ports. Further, at $Ri = 0.0$ and $Re = 100$, it is seen that a small recirculation cell is developed just at the top of the inlet port, due to increased inertia force. Further more, the size of the recirculation cell increases with increasing Re at the fixed Ri (= 0.0). The corresponding isotherm plots are presented in the right column of the Fig. 2. From the figures it it is clear that Reynolds number has significant effect on isotherms at the pure forced convection. Moreover, for Ri = 1.0 and different values of Re (= 50, 100, 150 and 200), it is seen from the figure 3 that the natural convection effect is

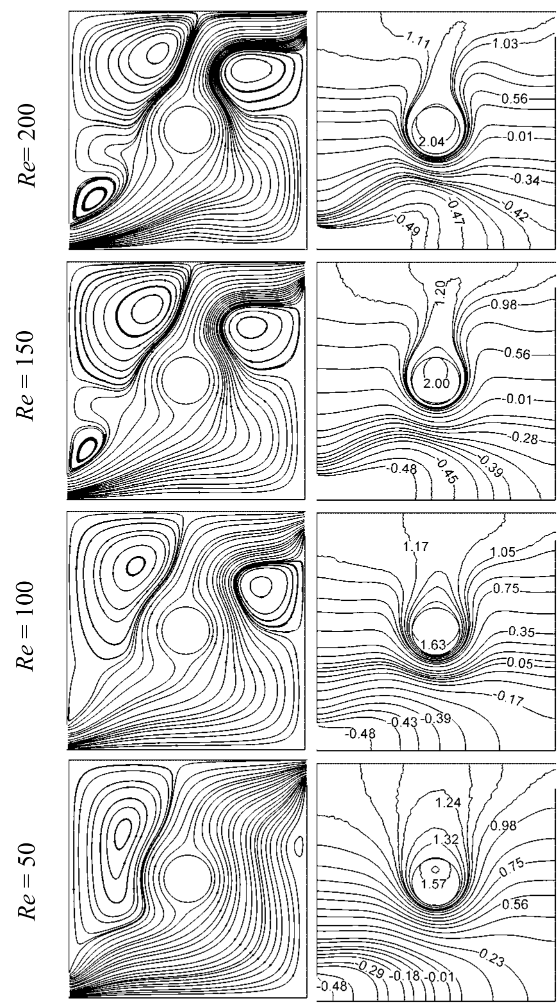


Fig. 4: Streamlines and Isotherms for different values Re, at $Ri = 5.0$.

present, but remains relatively weak at the higher values of Re. Further, increases of Ri gradually develops the size of the recirculation cell and leads to a large change in the streamline structures. Making a comparison of the isothermal lines for Ri = 1.0 and 5.0 and different selective values of Re with those for Ri = 0.0 and different selective values of Re, a significant difference is found as shown in the right column of the Figs 3-4.

The effect of the Reynolds number *Re* on the average Nusselt number and average fluid temperature in the cavity is shown in Fig. 5. It is noteworthy that the values of average Nusselt number decreases slowly with increasing Ri for the lower values of Re (= 50 and 100) and increases sharply in the forced convection dominated region and slowly in the free convection dominated region for the higher values of Re (= 150 and 200). Maximum values of Nu is found for the highest value of Re. On the other hand, average fluid temperature in the cavity decreases sharply with increasing *Re*, in the forced convection dominated region (Ri ≤ 0.5) and beyond these values of Ri it is decreases slowly for the higher values of Re. But the average fluid temperature in the cavity increases slowly with increasing Ri for the lowest value of Ri.

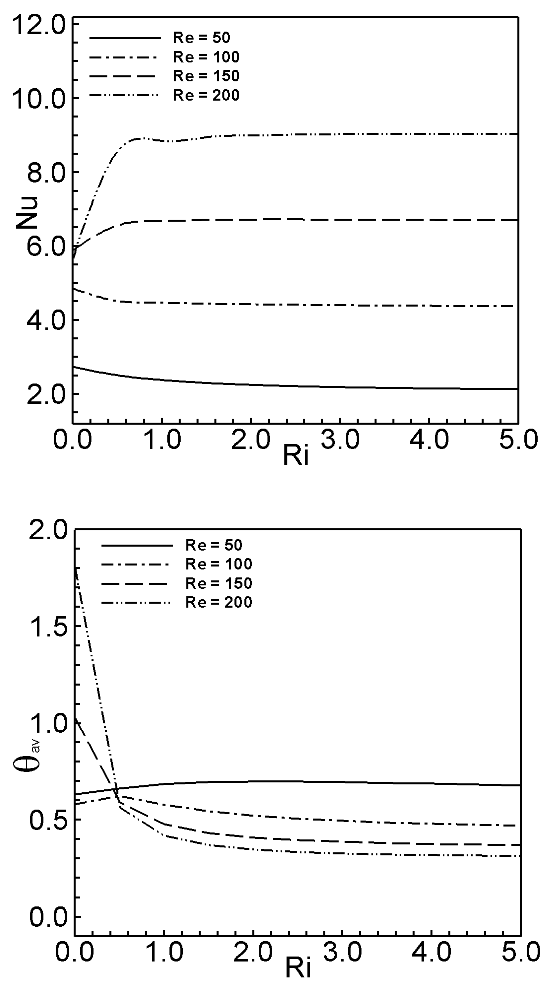


Fig. 5: Effect of Re on (i) average Nusselt number, (top) and (ii) average fluid temperature (bottom) in the cavity.

6. CONCLUSION

The present numerical investigation is made of mixed convection in an enclosure with a heat-generating horizontal circular body. Results are obtained for wide ranges of parameters *Re* and *Ri*. The following conclusions may be drawn from the present investigations:

Reynolds number has significant effect on the flow and thermal fields at the three convective regimes. Maximum average Nusselt number is found for the largest Reynolds number. On the other hand minimum average fluid temperature is found for Re = 100 in *Ri* ≤ 0.5, and for for Re = 200 in *Ri* > 0.5.

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8. NOMENCLATURE

Symbol	Meaning	Unit
d	cylinder diameter	(*m*)
g	gravitational acceleration	(ms^{-2})
Gr	Grashof number	
h	convective heat transfer coefficient	($Wm^{-2}K^{-1}$)
K_f	thermal conductivity of fluid	($Wm^{-1}K^{-1}$)
K_s	thermal conductivity of solid	($Wm^{-1}K^{-1}$)
K	solid fluid thermal conductivity ratio	
L	length of the cavity	(*m*)
Nu	Nusselt number	
p	dimensional pressure	(Nm^{-2})
P	dimensionless pressure	
Pr	Prandtl number	
Re	Reynolds number	
Ra	Rayleigh number	
Ri	Richardson number	
T	dimensional temperature	(*K*)
u, v	dimensional velocity components	(ms^{-1})
U, V	dimensionless velocity components	
$\bar{V}$	cavity volume	(m^3)
w	height of the opening	(*m*)
x, y	Cartesian coordinates	(*m*)
X, Y	dimensionless Cartesian coordinates	

Greek symbols

Symbol	Meaning	Unit
α	thermal diffusivity	(m^2s^{-1})
β	thermal expansion coefficient	(K^{-1})
ν	kinematic viscosity	(m^2s^{-1})
θ	non dimensional temperature	
ρ	density of the fluid	(kgm^{-3})

Subscripts

Symbol	Meaning	Unit
av	average	
h	heated wall	
i	inlet state	
c	cylinder center	

9. MAILING ADDRESS

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